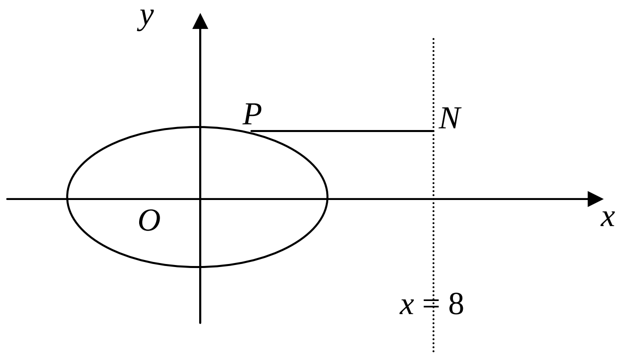
Mark Scheme (Results)

Summer 2013

GCE Further Pure Mathematics 3 (6669/01R)

Question Number	Scheme		Marks
	Foci (±5, 0), Directrices $x = \pm\frac{9}{5}$		
1.	$(\pm)ae = (\pm)5$ and $(\pm)\frac{a}{e} = (\pm)\frac{9}{5}$	Correct equations (ignore ±'s)	B1
	so $e = \frac{5}{a} \Rightarrow \frac{a^2}{5} = \frac{9}{5} \Rightarrow a^2 = 9$ or $a = \frac{5}{e} \Rightarrow \frac{5}{e^2} = \frac{9}{5} \Rightarrow e = \frac{5}{3} \Rightarrow a = 3$	M1: Solves using an appropriate method to find a^2 or a A1: $a^2 = 9$ or $a = (\pm)3$	M1A1
	$b^2 = a^2e^2 - a^2 \Rightarrow b^2 = 25 - 9$ so $b^2 = 16$ $(\Rightarrow b = 4)$ or $b^2 = a^2\left(e^2 - 1\right) \Rightarrow b^2 = 9\left(\frac{25}{9} - 1\right)$ $b^2 = 16$ $(\Rightarrow b = 4)$	M1: Use of $b^2 = a^2\left(e^2 - 1\right)$ to obtain a numerical value for b^2 or b A1: : $b^2 = 16$ or $b = (\pm)4$	M1 A1
	So $\frac{x^2}{9} - \frac{y^2}{16} = 1$	M1:Use of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ with their a^2 and b^2 A1: Correct hyperbola in any form.	M1 A1
			(7)

Question Number	Scheme		Marks
2.	l_1: $(\mathbf{i} - \mathbf{j} + \mathbf{k}) + \lambda(4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$ l_2: $(3\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}) + \lambda(-4\mathbf{i} + 6\mathbf{j} + \mathbf{k})$		
(a)	$\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 3 & 2 \\ -4 & 6 & 1 \end{vmatrix} = -9\mathbf{i} - 12\mathbf{j} + 36\mathbf{k}$	M1: Correct attempt at a vector product between $4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ and $-4\mathbf{i} + 6\mathbf{j} + \mathbf{k}$ (if the method is unclear then 2 components must be correct) allowing for the sign error in the *y* component. A1: Any multiple of $(3\mathbf{i} + 4\mathbf{j} - 12\mathbf{k})$	M1A1
			(2)
(b) Way 1	$\mathbf{a}_1 - \mathbf{a}_2 = \pm(2\mathbf{i} + 8\mathbf{j} + \mathbf{k})$	M1: Attempt to subtract position vectors A1: Correct vector $\pm(2\mathbf{i} + 8\mathbf{j} + \mathbf{k})$ (Allow as coordinates)	M1 A1
	So $p = \dfrac{\begin{pmatrix}2\\8\\1\end{pmatrix} \bullet \begin{pmatrix}-9\\-12\\36\end{pmatrix}}{\sqrt{9^2 + 12^2 + 36^2}}$	Correct formula for the distance using their vectors: $\dfrac{\text{"}\pm(2\mathbf{i} + 8\mathbf{j} + \mathbf{k})\text{"} \bullet \text{"}\mathbf{n}\text{"}}{\lvert\text{"}\mathbf{n}\text{"}\rvert}$	M1
	$p = \dfrac{\pm 78}{\sqrt{1521}} = \dfrac{\pm 78}{39} = 2$	M1: Correctly forms a scalar product in the numerator **and** Pythagoras in the denominator. **(Dependent on the previous method mark)** A1: 2 (**not** -2)	dM1 A1
			(5)
(b) Way 2	$(\mathbf{i} - \mathbf{j} + \mathbf{k}) \bullet (3\mathbf{i} + 4\mathbf{j} - 12\mathbf{k}) = -13\ (d_1)$ $(3\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}) \bullet (3\mathbf{i} + 4\mathbf{j} - 12\mathbf{k}) = 13\ (d_2)$	M1: Attempt scalar product between their **n** and either position vector A1: Both scalar products correct	M1A1
	$\dfrac{\pm 13}{\sqrt{3^2 + 4^2 + 12^2}} (= 1)$	Divides either of their scalar products by the magnitude of their normal vector. $\dfrac{d_1 \text{ or } d_2}{\lvert\text{"}\mathbf{n}\text{"}\rvert}$	M1
	$p = \dfrac{d_1}{\lvert\text{"}\mathbf{n}\text{"}\rvert} - \dfrac{d_2}{\lvert\text{"}\mathbf{n}\text{"}\rvert}$ or $2 \times \dfrac{d_1}{\lvert\text{"}\mathbf{n}\text{"}\rvert}$	M1: Correct attempt to find the required distance i.e. subtracts their $\dfrac{d_1}{\lvert\text{"}\mathbf{n}\text{"}\rvert}$ and $\dfrac{d_2}{\lvert\text{"}\mathbf{n}\text{"}\rvert}$ or doubles their $\dfrac{d_1}{\lvert\text{"}\mathbf{n}\text{"}\rvert}$ if $\lvert d_1 \rvert = \lvert d_2 \rvert$. **(Dependent on the previous method mark)** A1: 2 (**not** -2)	dM1 A1
			(5)
			Total 7

Question Number	Scheme		Marks
3. (a)		A closed curve approximately symmetrical about both axes. A vertical line to the right of the curve. A horizontal line from any point on the ellipse to the vertical line **with both P and N clearly marked.**	B1 (1)
3. (b)	M is $\left(\frac{x+8}{2}, y\right) = (X, Y)$ or $\left(\frac{6\cos\theta+8}{2}, 3\sin\theta\right) = (X, Y)$	M1: Finds the mid-point of PN A1: Correct mid-point	M1A1
	$\frac{(2X-8)^2}{36} + \frac{Y^2}{9} = 1$	M1: Attempt cartesian equation A1: Correct equation	M1 A1
			(4)
	The next 3 marks are dependent on having the equation of a circle.		
(c)	Circle because equation may be written $(x-4)^2 + y^2 = 3^2$	Convincing argument – allow follow through provided they do have a circle! Can be implied by their centre and radius.	B1ft
	The centre is (4, 0) and the radius is 3	M1: Use their circle equation to find centre and radius A1: Correct centre and radius	M1A1
			(3)
			Total 8
	<u>**Special Case:**</u> **In (b) they assume the locus is a circle and find the intercepts on the *x*-axis as (1, 0) and (7, 0) and hence deduce the centre (4, 0) and radius 3.** **This approach scores no marks in (b) but allow recovery in (c).**		

Question Number	Scheme		Marks
4.	$\begin{pmatrix} 2 & 0 & 3 \\ 0 & 2 & -1 \\ 0 & 1 & 2 \end{pmatrix}\begin{pmatrix} 1+s+t \\ -1+s+2t \\ 2 \quad -2t \end{pmatrix}$	M1: Writes Π_1 as a single vector A1: Correct statement	M1A1
	$\begin{pmatrix} 2 & 0 & 3 \\ 0 & 2 & -1 \\ 0 & 1 & 2 \end{pmatrix}\begin{pmatrix} 1+s+t \\ -1+s+2t \\ 2 \quad -2t \end{pmatrix} = \begin{pmatrix} 2+2s+2t+6-6t \\ -2+2s+4t-2+2t \\ -1+s+2t+4-4t \end{pmatrix}$		M1A1
	M1: Correct attempt to multiply A1: Correct vector in any form		
	$= \begin{pmatrix} 8+2s-4t \\ -4+2s+6t \\ 3+s-2t \end{pmatrix}$	Correct simplified vector	B1
	$\mathbf{r} = \begin{pmatrix} 8 \\ -4 \\ 3 \end{pmatrix} + s\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} + t\begin{pmatrix} -4 \\ 6 \\ -2 \end{pmatrix}$		
	$\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 2 & 1 \\ -4 & 6 & -2 \end{vmatrix} = -10\mathbf{i} + 20\mathbf{k}$	M1: Attempts cross product of their direction vectors A1: Any multiple of $-10\mathbf{i} + 20\mathbf{k}$	M1A1
	(8i – 4j + 3k).(i – 2k) = 8 - 6	Attempt scalar product of their normal vector with their position vector	M1
	r. (i - 2k) = 2	Correct equation (accept any correct equivalent e.g. **r. (-**10**i** + 20**k) =** -20)	A1
			(9)

Question Number	Scheme		Marks
5(a)	$I_n = \left[x^n(2x-1)^{\frac{1}{2}}\right]_1^5 - \int_1^5 nx^{n-1}(2x-1)^{\frac{1}{2}}\,\mathrm{d}x$	M1: Parts in the correct direction including a valid attempt to integrate $(2x-1)^{-\frac{1}{2}}$ A1: Fully correct application – may be un-simplified. (Ignore limits)	M1 A1
	$I_n = \underline{5^n \times 3 - 1} - \int_1^5 nx^{n-1}\underline{(2x-1)(2x-1)^{-\frac{1}{2}}\mathrm{d}x}$	Obtains a correct (possibly un-simplified) expression using the limits 5 and 1 **and** writes $(2x-1)^{\frac{1}{2}}$ as $(2x-1)(2x-1)^{-\frac{1}{2}}$	B1
	$I_n = 5^n \times 3 - 1 - 2nI_n + nI_{n-1}$	Replaces $\int x^n(2x-1)^{-\frac{1}{2}}\mathrm{d}x$ with I_n and $\int x^{n-1}(2x-1)^{-\frac{1}{2}}\mathrm{d}x$ with I_{n-1}	dM1
	$(2n+1)I_n = nI_{n-1} + 3\times 5^n - 1$ *	Correct completion to printed answer with no errors seen	A1cso
			(5)
(b)	$I_0 = \int_1^5 (2x-1)^{-\frac{1}{2}}\mathrm{d}x = \left[(2x-1)^{\frac{1}{2}}\right]_1^5 = 2$	$I_0 = 2$	B1
	$5I_2 = 2I_1 + 74$ and $3I_1 = I_0 + 14$	M1: Correctly applies the given reduction formula twice A1: Correct <u>equations</u> for I_2 and I_1 (may be implied)	M1 A1
	So $I_1 = \frac{16}{3}$ and $I_2 = \ldots$ or $5I_2 = 2\frac{I_0+14}{3} + 74$ and $I_2 = \ldots$	Completes to obtains a numerical expression for I_2	dM1
	$I_2 = \frac{254}{15}$		B1
			(5)
			Total 10

Question Number	Scheme		Marks
6. (a)	$\begin{pmatrix} 4 & 2 & 3 \\ 2 & b & 0 \\ a & 1 & 8 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 8 \\ \dots \\ \dots \end{pmatrix}, \ = \lambda\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \ \lambda = 8$	M1: Multiplies the given matrix by the given eigenvector M1: Equates the *x* value to λ A1: λ = 8	M1, M1, A1
			(3)
(b)	$\begin{pmatrix} 8 \\ 2+2b \\ a+2 \end{pmatrix} = "8"\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ So a = -2 and b = 7	M1: Their 2 + 2b = 2λ or their a + 2 = 0 A1: b = 7 **or** a = -2 A1: b = 7 **and** a = -2	M1 A1 A1
			(3)
(c)	$\begin{vmatrix} 4-\lambda & 2 & 3 \\ 2 & 7-\lambda & 0 \\ -2 & 1 & 8-\lambda \end{vmatrix} = 0$ $\therefore (4-\lambda)(7-\lambda)(8-\lambda) - 2\times 2(8-\lambda) + 3(2+2(7-\lambda)) = 0$		M1
	Correct attempt to establish the Characteristic Equation. = 0 is required but may be implied by later work Allow this mark if the equation is in terms of a, b, c		
	Attempts to factorise i.e. $(8-\lambda)(30-11\lambda+\lambda^2)$ or $(6-\lambda)(40-13\lambda+\lambda^2)$ or $(5-\lambda)(48-14\lambda+\lambda^2)$ (NB $240-118\lambda+19\lambda^2-\lambda^3=0$)		M1 A1
	M1: Attempt to factorise their cubic – an attempt to identify a linear factor and processes to obtain a simplified quadratic factor A1: Correct factorisation into one linear and one quadratic factor		
	Eigenvalues are 5 and 6	M1: Solves their equation to obtain the other eigenvalues A1: 5 and 6	M1 A1
			(5)
			Total 8

Question Number	Scheme		Marks
7(a)	Put 6cosh x = 9 - 2sinh x		M1
	$6\times\frac{1}{2}(e^{x}+e^{-x})=9-2\times\frac{1}{2}(e^{x}-e^{-x})$	Replaces cosh$x$ and sinh$x$ by the **correct** exponential forms	M1
	$4e^{x}+2e^{-x}-9=0 \Rightarrow 4e^{2x}-9e^{x}+2=0$	M1: Multiplies by e^{x} A1: Correct quadratic in e^{x} in any form with terms collected	M1 A1
	So $e^{x}=\frac{1}{4}$ or 2 and x = ln2 or $\ln\frac{1}{4}$	M1: Solves their quadratic in e^{x} A1: Correct values of x (Any correct equivalent form)	M1 A1
			(6)
(b)	Area is $\int(9-2\sinh x-6\cosh x)\,dx$	$\int(9-2\sinh x-6\cosh x)\,dx$ or $\int(6\cosh x-(9-2\sinh x))\,dx$ or the equivalent in exponential form	M1
	$\pm(9x-2\cosh x-6\sinh x)$ or $\pm(9x-4e^{x}+2e^{-x})$	M1: Attempt to integrate A1: Correct integration	M1 A1
	$\pm([9\ln 2-2\cosh\ln 2-6\sinh\ln 2]-[9\ln\frac{1}{4}-2\cosh\ln\frac{1}{4}-6\sinh\ln\frac{1}{4}])$		dM1
	Complete substitution of their limits from part (a). Depends on both previous M's		
	$=\pm(9\ln(2\div\frac{1}{4})-(e^{\ln 2}+e^{-\ln 2})-3(e^{\ln 2}-e^{-\ln 2})+(e^{\ln\frac{1}{4}}+e^{-\ln\frac{1}{4}})+3(e^{\ln\frac{1}{4}}-e^{-\ln\frac{1}{4}}))$		M1
	Combines logs correctly and uses cosh and sinh of ln correctly at least once		
	$\left(9\ln 8-\frac{5}{2}-\frac{18}{4}+4.25-11.25\right)$ =9ln8 –14 or 27ln2 – 14 Any correct equivalent		A1cao
	Subtracting the wrong way round could score 5/6 max		
			(6)
			Total 12
	<u>Note</u> **If they use $4e^{2x}-9e^{x}+2$ in (b) to find the area – no marks**		

Question Number	Scheme		Marks
8(a)	$\frac{dy}{dx} = x^{-\frac{1}{2}}$	Correct derivative (may be un-simplified)	B1
	$s = \int \sqrt{1+(x^{-\frac{1}{2}})^2}\,dx \;=\; \int_1^8 \sqrt{(1+\frac{1}{x})}\,dx$	A correct formula quoted or implied. There must be some working before the printed answer.	B1
			(2)
(b)	$x = \sinh^2 u \Rightarrow \frac{dx}{du} = 2\sinh u \cosh u$	Correct derivative	B1
	$(1+\frac{1}{x}) = 1 + \text{cosech}^2 u = \coth^2 u$	$(1+\frac{1}{x}) = \coth^2 u$ or $(1+\frac{1}{x}) = \frac{\cosh^2 u}{\sinh^2 u}$ (may be implied by later work)	B1
	$s = \int \coth u.2\sinh u \cosh u\,du =$ $\int 2\cosh^2 u\,du$	M1: Complete substitution A1: $\int 2\cosh^2 u\,du$	M1 A1
	$= u + \frac{1}{2}\sinh 2u$ or $\frac{1}{4}e^{2u} + u - \frac{1}{4}e^{-2u}$	M1: Uses $\cosh 2u = \pm 2\cosh^2 u \pm 1$ or changes to exponentials in an attempt to integrate an expression of the form $k\cosh^2 u$ A1: Correct integration	dM1 A1
	$x = 8 \Rightarrow u = \text{arsinh}\sqrt{8} = \ln(3+2\sqrt{2})$, $x = 1 \Rightarrow u = \text{arsinh}1 = \ln(1+\sqrt{2})$		
	$\left[u + \frac{1}{2}\sinh 2u\right]_{\text{arsinh}1}^{\text{arsinh}\sqrt{8}}$ $= \text{arsinh}\sqrt{8} + \frac{1}{2}\sinh(2\text{arsinh}\sqrt{8}) - (\text{arsinh}1 + \frac{1}{2}\sinh(2\text{arsinh}1))$ *or* $\left[\frac{1}{4}e^{2u} + u - \frac{1}{4}e^{-2u}\right]_{\text{arsinh}1}^{\text{arsinh}\sqrt{8}}$ $= \frac{1}{4}e^{\text{arsinh}\sqrt{8}} + \text{arsinh}\sqrt{8} - \frac{1}{4}e^{\text{-2arsinh1}}$ *or* $\left[\text{arsinh}\sqrt{x} + \frac{1}{2}\sinh(2\text{arsinh}\sqrt{x})\right]_1^8$ $= \text{arsinh}\sqrt{8} + \frac{1}{2}\sinh(2\text{arsinh}\sqrt{8}) - (\text{arsinh}1 + \frac{1}{2}\sinh(2\text{arsinh}1))$		ddM1A1
	M1: The limits $\text{arsinh}\sqrt{8}$ and $\text{arsinh}1$ or their $\ln(3+2\sqrt{2})$ and $\ln(1+\sqrt{2})$ used correctly in their f(u) or the limits 8 and 1 used correctly if they revert to x **Dependent on both previous M's** A1: A completely correct expression		
	$\ln(1+\sqrt{2}) + 5\sqrt{2}$		A1
			(9)
			Total 11